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**On Some Impossibility Theorems in Population Ethics**

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At least since Derek Parfit presented his “mere addition paradox”,[[1]](#footnote-1) the difficulties of formulating a viable population axiology have been widely recognized. Building on Parfit’s work, a number of philosophers have proved impossibility theorems, showing that certain plausible adequacy conditions are mutually inconsistent.[[2]](#footnote-2) To the best of my knowledge, the most impressive and important such results are due to Gustaf Arrhenius.[[3]](#footnote-3) His theorems involve more compelling adequacy conditions and weaker assumptions of measurement than earlier work in this area. On the basis of these theorems, Arrhenius is inclined to deny the existence of a satisfactory population axiology.[[4]](#footnote-4)

 The aim of this paper is to show that Arrhenius’s impossibility results are not inescapable. I shall mainly focus on his “sixth” theorem, which he considers to be his strongest result. Arrhenius’s proof of this theorem requires a certain assumption, as regards the order of welfare levels, which is more contentious than he recognizes. This assumption rules out “non-Archimedean” theories of welfare. If such theories are not excluded, there are, as I shall show, population axiologies that satisfy all the adequacy conditions of Arrhenius’s sixth theorem. In the penultimate section of the paper I shall argue, moreover, that my objection pertains to all of Arrhenius’s axiological impossibility theorems.[[5]](#footnote-5) Since non-Archimedean theories of welfare are far from obviously false, Arrhenius’s results fail to show that there is no acceptable population axiology.

**1. Arrhenius’s Sixth Impossibility Theorem and the Crucial Assumption**

Arrhenius’s sixth theorem states that no population axiology satisfies five adequacy conditions, which are informally rendered as follows:

*Egalitarian Dominance*: If population *A* is a perfectly equal population of the same size as population *B*, and every person in *A* has higher welfare than every person in *B*, then *A* is better than *B*, other things being equal.

*General Non-Extreme Priority:* For any welfare level **A** and any population *X*, there is a number *n* of lives such that a population consisting of the *X*-lives, *n* lives with very high welfare, and one life with welfare **A**, is at least as good as a population consisting of the *X*-lives, *n* lives with very low positive welfare, and one life with welfare slightly above **A**, other things being equal.

*Non-Elitism*: For any triplet of welfare levels **A**, **B**, and **C**, **A** slightly higher than **B**, and **B** higher than **C**, and for any one-life population *A* with welfare **A**, there is a population *C* with welfare **C**, and a population *B* of the same size as *A*∪*C* and with welfare **B**, such that for any population *X* consisting of lives with welfare ranging from **C** to **A**, *B*∪*X* is at least as good as *A*∪*C*∪*X*, other things being equal.

*Weak Non-Sadism*: There is a negative welfare level and a number of lives at this level such that [for any population *X*] an addition of any number of people with positive welfare [to *X*] is at least as good as an addition of the lives with negative welfare [to *X*], other things being equal.

*Weak Quality Addition*: For any population *X*, there is a perfectly equal population with very high positive welfare, a very negative welfare level, and a number *n* of lives at this level, such that the addition of the high welfare population to *X* is at least as good as the addition of any population consisting of the *n* lives with negative welfare and any number of lives with very low positive welfare to *X*, other things being equal.[[6]](#footnote-6)

Although some of these conditions may not be entirely perspicuous, I think they are, on reflection, very plausible. In any case, they will not be questioned here.

 Arrhenius’s proof proceeds by formulating more exact (and more technical) versions of these conditions, and showing that these exact conditions are mutually inconsistent. He thus arrives at

*The Sixth Impossibility Theorem:* There is no population axiology which satisfies Egalitarian Dominance, General Non-Extreme Priority, Non-Elitism, Weak Non-Sadism, and Weak Quality Addition.[[7]](#footnote-7)

 The main background assumptions of this theorem are as follows. A *life* is individuated by the person whose life it is and the kind of life it is. A *population* is a finite set of lives in a possible world. A *population axiology* is a quasi-order of all possible populations, with respect to their value. (A *quasi-order* is a reflexive and transitive, but not necessarily complete relation.) The relation “*has at least as high welfare as*”, denoted ≿, quasi-orders the set *L* of possible lives. The relations “has higher welfare than”, denoted ≻, and “has equally high welfare as”, denoted ~, are the asymmetric and the symmetric parts, respectively, of (*L*, ≿). Some possible lives have *positive* welfare, some have *negative* welfare, and some have *neutral* welfare. Which of these categories a life belongs to is independent of scales of measurement. We assume that *L* includes a wide range of lives, from ones with very high positive welfare to ones with very low negative welfare. Further, a *welfare level* is an equivalence class of *L* under ~. Let **L** be the set of welfare levels, let (**L**, **≿**)be the ~-reduction of (*L*, ≿), and let **≻** denote the asymmetric part of **≿**.[[8]](#footnote-8)

 By assuming only that (*L*, ≿) is a quasi-order, Arrhenius leaves open the possibility that some lives, and the corresponding welfare levels, are incomparable. Thus, there may be welfare levels **X** and **Y**, such that neither **X ≿ Y** nor **Y** **≿** **X**. In order to simplify the discussion, I shall mostly ignore this possibility, and assume that (**L**, **≿**) is a complete order. If the sixth impossibility theorem can be shown to be escapable, assuming that (**L**, **≿**) is complete, it is *a fortiori* escapable if (**L**, **≿**) is allowed to be incomplete.

 A final assumption that Arrhenius makes is crucial for the purposes of this paper. This assumption is somewhat complex, and not very precisely stated by Arrhenius. First, he presumes that (**L**, **≿**) is either “discrete” or “dense”, according to the following definitions:

*Discreteness:* For any pair of welfare levels **X** and **Y**, there are only finitely many welfare levels between **X** and **Y**.[[9]](#footnote-9)

*Denseness:* There is a welfare level between any pair of distinct welfare levels.[[10]](#footnote-10)

Let us refer to this notion of discreteness as *Arrhenius discreteness*, or *A-discreteness*, for short. Second, Arrhenius claims that (**L**, **≿**) is “fine-grained”. An A-discrete order of welfare levels is taken to be fine-grained iff the difference between any two adjacent levels is “merely slight”, in an undefined, intuitive sense.[[11]](#footnote-11) (How fine-grainedness should be understood when it comes to dense orders will be discussed in the next section.) Third, he claims that if (**L**, **≿**) is fine-grained and dense, it has a fine-grained A-discrete suborder which is comprehensive, in the sense that for any **X** in (**L**, **≿**), there is a **Y** in the suborder, such that the difference between **X** and **Y** is merely slight.[[12]](#footnote-12) In what follows, “suborder” refers to such comprehensive suborders.

 These three claims together imply

*The Crucial Assumption:* (**L**, **≿**) has a fine-grained A-discrete suborder.[[13]](#footnote-13)

 This assumption is necessary for Arrhenius’s proof of the sixth theorem, since the proof requires that welfare levels can be represented by integers. There must be an integer-valued function *f*, such that, for any welfare levels **X** and **Y**, *f*(**X**) ≥ *f*(**Y**) iff **X ≿** **Y**.[[14]](#footnote-14) Integer representability presupposes A-discreteness, since there are only finitely many integers between any given pair of integers. Arrhenius’s proof shows that no population axiology satisfies the exact versions of his five adequacy conditions, given that the order of welfare levels assumed in the proof is fine-grained and A-discrete. The Crucial Assumption implies that, even if (**L**, **≿**) is not itself A-discrete, the sixth theorem can be proved relative to an A-discrete suborder of (**L**, **≿**). This suffices to establish the theorem, since for a population axiology to quasi-order the set of finite subsets of *L*, it must quasi-order every subset of this set.

 Suppose, to the contrary, that the Crucial Assumption does not hold. That is, no A-discrete suborder of (**L**, **≿**) is fine-grained. This implies that every A-discrete suborder (**L**\*, **≿**\*) contains at least one pair of adjacent welfare levels **X** and **Y**, such that **X ≻\* Y**, and **X** is not merely slightly higher than **Y**. In Arrhenius’s exact version of General Non-Extreme Priority, the locution “one life with welfare slightly above **A**” is substituted by the assumption that the one life is at the higher welfare level adjacent to **A**. Similarly, in the exact version of Non-Elitism, the assumption that **A** is “slightly higher” than **B** is replaced by the assumption that **A** is higher than and adjacent to **B**. Hence, these exact conditions, on which Arrhenius’s proof is based, do not reflect the content of the informal conditions, and fail to be intuitively compelling, unless (**L**\*, **≿**\*) is fine-grained.

 The Crucial Assumption is thus essential for Arrhenius’s purposes. I shall argue that this assumption is insufficiently supported by Arrhenius, and that there are positive reasons for doubting it.

**2. Denseness, Fine-grainedness, and Two Notions of Discreteness**

The above criterion for fine-grainedness, in terms of adjacent welfare levels, is not applicable to dense orders, since such orders do not contain any pairs of adjacent elements. Arrhenius does not say what fine-grainedness amounts to, as regards dense orders. He argues, however, that all dense orders have a fine-grained A-discrete suborder:

If Denseness is true of [(**L**, **≿**)], then we can form a [suborder (**L\***, **≿\***) of (**L**, **≿**)] such that [A-discreteness] is true of [(**L\***, **≿\***)], and such that all the conditions that are intuitively plausible in regard to populations which are subsets of *L* are also intuitively plausible in regard to populations which are subsets of *L\**. Given that Denseness is true of [(**L**, **≿**)], one cannot plausibly deny that there is such a [suborder (**L\***, **≿\***)] since the order of the welfare levels in [(**L\***, **≿\***)] could be arbitrarily fine-grained even though [A-discreteness] is true of [(**L\***, **≿\***)].[[15]](#footnote-15)

 This argument is faulty. Suppose, for example, that (**L**, **≿**) is isomorphic to the natural order of the real numbers strictly smaller than 0, followed by the real numbers strictly greater than 10. Suppose also that differences between numbers mirror differences between corresponding welfare levels, and that a difference represented by a numerical difference of 10 or more is not merely slight. Under these assumptions, (**L**, **≿**) is dense, but lacks a fine-grained A-discrete suborder. Any A-discrete suborder will include a “gap”, in the form of a welfare level represented by a number smaller than 0, immediately followed by a level represented by a number greater than 10.

 Intuitively, the dense order just discussed is not fine-grained. This is because there is a “hole” between welfare levels represented by numbers smaller than 0, and levels represented by numbers greater than 10.[[16]](#footnote-16) Hence, it might still be true that every *fine-grained* dense order has a fine-grained A-discrete suborder. An obvious suggestion is that a dense order is fine-grained if it lacks holes. Figuratively speaking, a dense order without holes is like an unbroken line, with each element of the order corresponding to a point on the line. This seems to constitute a high degree of fine-grainedness. Absence of holes, in a dense order, is equivalent to “Dedekind completeness”.[[17]](#footnote-17) Applied to (**L**, **≿**), this amounts to the following. An *upper bound* of a [suborder](http://en.wikipedia.org/wiki/Subset) (**L\***, **≿\***) of (**L**, **≿**) is a welfare level in (**L**, **≿**) which is [at](http://en.wikipedia.org/wiki/Greater_than_or_equal_to) least as high as every level in (**L\***, **≿\***). A *least upper bound* of (**L\***, **≿\***) is an upper bound of (**L\***, **≿\***) that is at least as low as every upper bound of (**L\***, **≿\***). (**L**, **≿**) is *Dedekind complete* iff every suborder of (**L**, **≿**) with an upper bound has a least upper bound.

 Let us, then, make the plausible assumption that all Dedekind complete dense orders should be classified as fine-grained.[[18]](#footnote-18) This puts us in a position to determine whether the Crucial Assumption follows from Arrhenius’s supposition that (**L**, **≿**) is either fine-grained and A-discrete, or fine-grained and dense. The answer is negative. To see this, suppose that (**L**, **≿**) is isomorphic to the Cartesian product of the real numbers, *Re* × *Re*, ordered lexicographically. That is, an ordered pair (*x*, *y*) of realsis at least as great as an ordered pair (*z*, *w*) iff *x* > *z*, or *x* = *z* and *y* ≥ *w*. This means that (**L**, **≿**) is dense and Dedekind complete.[[19]](#footnote-19) Suppose, further, that the difference between two welfare levels, **X** and **Y**, is merely slight, in the relevant intuitive sense, only if the first number in the ordered pair representing **X** is the same as the first number in the pair representing **Y**. This implies that the Crucial Assumption does not hold. That is, (**L**, **≿**) has no fine-grained A-discrete suborder. For example, any A-discrete suborder will contain a highest level among those represented by an ordered pair (1, *x*), for some *x*, immediately followed by a level represented by an ordered pair (*y*, *z*), *y* > 1. The suborder thus has a gap at this point, in the sense of two adjacent levels with a not merely slight difference.[[20]](#footnote-20)

 Arrhenius might respond that denseness is an unlikely property of (**L**, **≿**), anyway. In fact, he regards A-discreteness as more probable than denseness:

My own inclination is that [A-discreteness] rather than Denseness is true. If the latter is true, then for any two lives *p*1 and *p*2, *p*1 with higher welfare than *p*2, there is a life *p*3 with welfare in between *p*1 and *p*2, and a life *p*4 with welfare in between *p*3 and *p*2, and so on *ad infinitum*. It is improbable, I think, that there [is] such fine discrimination between the welfare of lives, even in principle. Rather, what we will find at the end of such a sequence of lives is a pair of lives in between which we cannot find any life or only lives with roughly the same welfare as both of them.[[21]](#footnote-21)

 This argument disregards the possibility that (**L**, **≿**) is neither A-discrete nor dense. In particular, Arrhenius does not distinguish between A-discreteness and a logically weaker property that we may call *standard discreteness*, or *S-discreteness*, for short:

*S-discreteness:* For any non-maximal (non-minimal) welfare level **X**, there is a higher (lower) welfare level **Y**, such that there is no welfare level between **X** and **Y**.[[22]](#footnote-22)

Arrhenius’s argument from limited discrimination does not support A-discreteness in particular. At most, it supports S-discreteness.[[23]](#footnote-23)

 To see that S-discreteness does not imply A-discreteness, consider the Cartesian product of the integers, *I* × *I*, ordered lexicographically. This order obviously satisfies the analogue to S-discreteness. For any ordered pair *P*, there is a greater ordered pair *P\**, such that there is no ordered pair between *P* and *P\**. For example, (1, 2) is greater than (1, 1), and there is no ordered pair between these two. The analogue to A-discreteness, on the other hand, is not satisfied. For instance, there is an infinite number of ordered pairs between (1, 1) and (2, 1), viz., the pairs (1, 2), (1, 3), (1, 4) …., followed by the pairs … (2, -2), (2, -1), (2, 0).

 What, then, if we suppose (**L**, **≿**) to be fine-grained and S-discrete, but not necessarily A-discrete? Presumably, the same criterion for fine-grainedness applies, as in the case of A-discreteness. That is, (**L**, **≿**) is fine-grained iff the difference between any two adjacent levels is merely slight. If so, fine-grainedness and S-discreteness of (**L**, **≿**) do not imply the Crucial Assumption. Suppose that (**L**, **≿**) is isomorphic to the lexicographic order of *I* × *I*, and, again, that the difference between **X** and **Y** is merely slight only if the first number in the ordered pair representing **X** is the same as the first number in the pair representing **Y**. This implies that any A-discrete suborder has a gap.

 To sum up, Arrhenius fails to support the assumption that (**L**, **≿**) is either A-discrete or dense, since he does not consider other possibilities. Further, his argument against denseness does not support A-discreteness, but only S-discreteness. Fine-grainedness and S-discreteness do not imply the Crucial Assumption. Moreover, he is wrong in claiming that the Crucial Assumption follows from denseness and fine-grainedness. Hence, the Crucial Assumption is poorly defended.[[24]](#footnote-24)

**3. Archimedeanness and the Crucial Assumption**

The Crucial Assumption may of course be true, even though Arrhenius’s arguments in its favour are weak. The question, then, is whether there are any positive reasons to doubt its truth. I think that there are, in fact, such reasons. The Crucial Assumption immediately implies the following principle:

*Proto-Archimedeanness:* For any welfare levels **X** and **Y** in (**L**, **≿**), such that **X** **≻** **Y**, there is a finite sequence of welfare levels **V***n* **≻** ... **≻** **V**1 in (**L**, **≿**), such that **V***n* **≿** **X**, **Y** **≿ V**1, and, for each pair **V***i*, **V***i-*1, 1 < *i* ≤ *n*, the difference between **V***i* and **V***i-*1 is merely slight.

That is, the difference between any two welfare levels is bridged in a finite number of small steps.

 Moreover, a stronger condition follows if a difference is taken to be merely slight only if it is not significantly greater than some other difference between welfare levels in (**L**, **≿**). It appears that Arrhenius’s criterion for fine-grainedness, as regards A-discrete orders, presupposes such an interpretation of “merely slight”.[[25]](#footnote-25) An A-discrete order is not sufficiently fine-grained to be used in Arrhenius’s proof, if some differences between adjacent levels are much greater than other such differences. Given that all merely slight differences are roughly equal in size,[[26]](#footnote-26) the Crucial Assumption implies

*Archimedeanness:* For any welfare levels **X**, **Y**, **Z**, and **U** in (**L**, **≿**), such that **X** **≻** **Y** and **Z** **≻** **U**, there is a finite sequence of welfare levels **V***n* **≻** ... **≻** **V**1 in (**L**, **≿**), such that **V***n* **≿** **X**, **Y** **≿ V**1, and, for each pair **V***i*, **V***i-*1, 1 < *i* ≤ *n*, the difference between **V***i* and **V***i-*1 is roughly equal to the difference between **Z** and **U**.

That is, the difference between any two welfare levels is bridged in a finite number of *arbitrarily* small steps. [[27]](#footnote-27)

 Many philosophers have expressed views which are incompatible with Archimedeanness, and arguably also with proto-Archimedeanness. Parfit claims that a “century of ecstasy”, 100 years of extremely high quality of life, is better than a “drab eternity”, an infi­nitely long life containing nothing bad, and nothing good except muzak and potatoes. “Though each day of the Drab Eternity would have some [constant] value for me, *no* amount of this value could be as good for me as the Century of Ecstasy.”[[28]](#footnote-28) This view contradicts Archimedeanness, since it implies that the welfare difference between the century of ecstasy and, say, a life consisting of one day of muzak and potatoes cannot be bridged by adding a finite number of days of muzak and potatoes to the latter life, although each such addition brings a constant increase in welfare level.[[29]](#footnote-29)

 In a similar vein, James Griffin suggests that there are examples of

a positive value that, no matter how often a certain amount is added to itself, cannot become greater than another positive value, and cannot, not because with piling up we get diminishing value [...] , but because they are the sort of value that, even remaining constant, cannot add up to some other value.[[30]](#footnote-30)

Thus, Griffin finds it

plausible that, say, fifty years at a very high level of well-being—say, the level which makes possible satisfying personal relations, some understanding of what makes life worth while, appreciation of great beauty, the chance to accomplish something with one’s life—outranks any number of years at the level just barely worth living—say, the level at which none of the former values are possible and one is left with just enough surplus of simple pleasureover pain to go on with it.[[31]](#footnote-31)

 Arrhenius is, of course, well aware that many philosophers hold views like those just cited.[[32]](#footnote-32) Nevertheless, he does not seem to recognize the inconsistency between such views and the Crucial Assumption. I suspect that this is partly due to the fact that he does not distinguish between A-discreteness and S-discreteness.

**4. A Non-Archimedean Toy Theory of Welfare**

In order to construct a simple theory of welfare that satisfies fine-grainedness and S-discreteness, but violates the Crucial Assumption, we may suppose that (**L**, **≿**) is isomorphic to the lexicographic order of *I* × *I*, and that all differences between adjacent levels are merely slight. There are, let us imagine, two kinds of welfare components. One of these kinds is “superior” to the other, in the sense that any amount of the superior positive (negative) components contributes more to the total positive (negative) welfare in a life than any amount of the “inferior” positive (negative) components. Assume, further, that superior and inferior welfare is measured on separate additive ratio scales. Any welfare level can then be represented by an ordered pair of integers, (*h*, *l*), where *h* (for “higher”) represents a net amount of superior welfare, and *l* (for “lower”) represents a net amount of inferior welfare. We make the following assumptions:

(1) A welfare level **X**, represented by (*h***X**, *l***X**), is *at least as high as* a welfare level **Y**, represented by (*h***Y**, *l***Y**), iff *h***X** > *h***Y**, or *h***X** = *h***Y** and *l***X** ≥ *l***Y**.

(2) A welfare level **X**, represented by (*h***X**, *l***X**), is

*positive* iff *h***X** > 0, or *h***X** = 0 and *l***X** > 0,

*negative* iff *h***X** < 0, or *h***X** = 0 and *l***X** < 0,

*neutral* iff *h***X** = 0 and *l***X** = 0,

*very high* iff *h***X** ≥ *m*, for a particular positive integer *m*,

*very low but positive* only if *h***X** = 0 and *l***X** > 0,

*slightly negative* only if *h***X** = 0 and *l***X** < 0, and

*very negative* iff *h***X** ≤ *n*, for a particular negative integer *n*.

(3) A welfare level **X**, represented by (*h***X**, *l***X**), is *merely slightly higher than* a welfare level **Y**, represented by(*h***Y**, *l***Y**), only if *h***X** = *h***Y** and *l***X** = *l***Y** + *r*, *r* > 0.

 Under these assumptions, the Crucial Assumption is violated. Any A-discrete suborder of (**L**, **≿**) contains some pair of adjacent welfare levels **X** and **Y**, such that **X** is very high, whereas **Y** is not. By (3), therefore, the difference between **X** and **Y** is not merely slight. Similarly, any A-discrete suborder of (**L**, **≿**) contains a pair of adjacent levels **Z** and **U**, such that **U** is very negative, while **Z** is not. Again, (3) implies that the difference between **Z** and **U** is not merely slight.

 Moreover, it is easy to see that S-discreteness of (**L**, **≿**) is not an essential feature of this example. We may instead suppose that (**L**, **≿**) is isomorphic to *Re* × *Re*, and hence dense and fine-grained, but retain the other assumptions of the theory. The Crucial Assumption is violated in this case, as well.

**5. A Counterexample** **to the Sixth Impossibility Theorem**

What if the Crucial Assumption is weakened, to merely state that (**L**, **≿**) has a fine-grained S-discrete suborder? This assumption is trivial, given that (**L**, **≿**) is itself fine-grained, since the differences between adjacent levels in an S-discrete suborder of (**L**, **≿**) can, indeed, be arbitrarily small. However, it is not difficult to show that the sixth impossibility theorem is false if the Crucial Assumption is thus weakened.

 To this end, let us use our toy theory of welfare, in either its S-discrete or its dense variant. Given fine-grainedness, the weakened version of the Crucial Assumption is satisfied. Further, we assume that superior as well as inferior welfare can be added across lives. Thus, letting (*hA*, *lA*) and (*hB*, *lB*) represent the total amounts of superior and inferior welfare in populations *A* and *B*, respectively, *A* contains more welfare than *B* iff *hA* > *hB*, or *hA* = *hB* and *lA* > *lB*. Finally, we assume that a population is, other things equal, better than another iff it contains more welfare.[[33]](#footnote-33)

 Let us verify that this population axiology, which we may label “non-Archimedean totalism”, satisfies all of Arrhenius’s adequacy conditions. That the axiology satisfies Egalitarian Dominance and Weak Non-Sadism is obvious. To see that Weak Quality Addition is satisfied we need only note that adding any number of lives with very high welfare to a population *X* always makes the resulting population better than *X*, according to our axiology, whereas adding lives with very negative welfare plus any number of lives with very low positive welfare always makes the resulting population worse than *X*. Thus, let (*hX*, *lX*) be the total welfare in *X*. An addition of the first kind means adding (*m*, *n*), *m* > 0, to (*hX*, *lX*), while an addition of the second kind means adding (*r*, *s*), *r* < 0. No matter what the values of *n* and *s* are, (*hX* + *m*, *lX* + *n*) is greater than (*hX* + *r*, *lX* + *s*).

 General Non-Extreme Priority is also satisfied. In fact, the condition holds for any number *n* of lives added to population *X*. Let (*hX*, *lX*) be the total welfare in *X* and let welfare level **A** be represented by (*h***A**, *l***A**). Adding some life or lives with very high welfare and one life at level **A** to *X* yields (*hX* + *h***A** + *m*, *lX* + *l***A** + *r*), *m* > 0. Adding some life or lives with very low positive welfare and one life slightly above **A** to *X* yields (*hX* + *h***A**, *lX* + *l***A** + *u* + *s*), *u* > 0. Irrespective of the values of *r*, *s* and *u*, (*hX* + *h***A** + *m*, *lX* + *l***A** + *r*) is greater than (*hX* + *h***A**, *lX* + *l***A** + *u* + *s*).

 It remains to consider Non-Elitism. Let level **A** be represented by (*h***A**, *l***A**), implying that level **B** is represented by (*h***A**, *l***A** - *r*), *r* > 0. Also, let level **C** be represented by(*h***C**, *l***C**), and let (*hX*, *lX*) be the total welfare in population *X*. Let *n* be the number of people in populations *B* and *A*∪*C*. According to our axiology, *B*∪*X* is at least as good as *A*∪*C*∪*X* iff (*nh***A** + *hX*, *n*[*l***A** – *r*] + *lX*) is at least as great as (*h***A** + [*n –* 1]*h***C** + *hX*, *l***A**+ [*n* – 1]*l***C** + *lX*). Cancelling out the terms *hX* and *lX*,this is equivalent to (*nh***A**, *n*[*l***A** – *r*]) being at least as great as (*h***A** + [*n* – 1]*h***C**, *l***A**+ [*n* -1]*l***C**). Since *h***A** > *h***C**, or *h***A** = *h***C** and *l***A**– *r* > *l***C**, this holds for any *n*.

 Thus, if we assume a non-Archimedean theory of welfare, there are population axiologies which satisfy all of Arrhenius’s adequacy conditions. Moreover, such an axiology can, as in the case of non-Archimedean totalism, conform to the simple and intuitively appealing idea that a population is, other things equal, better than another iff it contains more welfare.

**6. The Plausibility of Non-Archimedean Theories of Welfare**

The non-Archimedean toy theory of welfare outlined in section 4 is rather simplistic. First, it assumes that all welfare levels are comparable. Second, the assumption that there are two kinds of welfare components, such that the smallest amount of the superior kind trumps any amount of the inferior kind is perhaps not very plausible.[[34]](#footnote-34) However, a non-Archimedean theory can be much more sophisticated than our toy model. First, it can allow for incomparability between welfare levels. Second, non-Archimedeanness need not be a simple matter of some welfare components being superior to others. Although the representation of welfare levels by ordered pairs of numbers may naturally suggest such a simple superiority view,[[35]](#footnote-35) this type of representation could be used also for other kinds of non-Archimedean theories. Conversely, non-Archimedean theories, including superiority views, can be mathematically represented in other ways.

 On a more complex view, non-Archimedeanness may be a holistic effect, arising from the combination of different welfare components, none of which is in itself superior. To illustrate this possibility, let us assume an “objective list” theory of welfare, akin to the one suggested by Griffin. According to this theory, pleasure, knowledge, friendship, freedom, appreciation of beauty, development of one’s talents, purposeful activity, and so on, are positive welfare components. It seems plausible to claim that for a life to have very high welfare, it must contain all or most of these things, to some degree. A life containing just one or two types of welfare components will likely be of an impoverished kind, having at best moderately high welfare. Nevertheless, it appears that an impoverished life can always be slightly improved, by adding more welfare components of the type or types already included in the life. However, such additions can never result in a life with very high welfare. Any finite series of improvements that takes us from an impoverished life to a life with very high welfare must include at least one step at which a welfare component of a new type is added. Such a step, arguably, means an improvement that cannot be arbitrarily slight.

 Obviously, much work is needed in order to furnish the details of such a holistic non-Archimedean theory of welfare. But the general idea has enough plausibility, I believe, to cast doubt on Archimedeanness. Consider a life, *p*1, containing no positive or negative welfare components except one second of mild pleasure. Compared to *p*1, a life, *p*2, containing nothing but two seconds of the same kind of mild pleasure, seems slightly better. And so on. Thus, each such life, *pi*, *i* = 1, 2, 3, … is a representative of a welfare level **X***i*, such that **X***i*+1 **≻** **X***i*, for all *i* > 1. Let us also suppose that the difference between adjacent levels **X***i*+1 and **X***i* is the same, no matter the value of *i*. That is, an added second of mild pleasure has constant marginal value. Further, let *p\** be the best life you can imagine, and let **X***\** be its welfare level. Probably, *p\** contains welfare components of many different kinds, interrelated in diverse ways. It is very plausible to claim that **X***\** **≻** **X***i*, for any *i*. However, if Archimedeanness is true, the difference between **X\*** and **X**1 is bridged by a finite number of differences, each roughly equal to the difference between **X***i*+1 and **X***i*. Hence, it follows that **X***j* **≿** **X\***, for some positive integer *j*.

The assumption of constant marginal value may be questioned. Perhaps the difference between adjacent levels **X***i*+1 and **X***i* diminishes as *i* increases. However, the conclusion that **X***j* **≿** **X\***, for some *j*, still follows from Archimedeanness, as long as there is, for any *i*, some *k* > *i*, such that the difference between **X***k* and **X***i* equals that between **X**2 and **X**1.[[36]](#footnote-36) To deny that there is such a *k* is to claim that if a life contains a sufficient number of seconds of mild pleasure (and no other welfare components), *no* extra number of such seconds yields an improvement, in terms of welfare, equal to the improvement from one to two seconds of mild pleasure. This claim does not appear very plausible.

 At the very least, I think we may conclude that Archimedeanness is not so evidently true that it can legitimately be presupposed, as a background assumption, in an impossibility theorem purporting to establish the non-existence of an acceptable population axiology.

**7. Arrhenius’s Other Impossibility Theorems**

The Crucial Assumption is a background assumption for each of Arrhenius’s six axiological impossibility theorems.[[37]](#footnote-37) If the Crucial Assumption is replaced by the weaker assumption that (**L**, **≿**) has a fine-grained S-discrete suborder, non-Archimedean totalism constitutes a counter-example also to Arrhenius’s first, fourth, and fifth impossibility theorems. (This is shown in the appendix.) His second and third theorems, on the other hand, include one adequacy condition that is violated by non-Archimedean totalism. This condition is as follows:

*Inequality Aversion:* For any triplet of welfare levels **A**, **B**, and **C**, **A** higher than **B**, and **B** higher than **C**, and for any population *A* with welfare **A**, there is a larger population *C* with welfare **C**, such that a perfectly equal population *B* of the same size as *A*∪*C* and with welfare **B**, is at least as good as *A*∪*C*, other things being equal.[[38]](#footnote-38)

 Non-Archimedean totalism violates this condition if, for example, **A** is a very high welfare level, while **B** and **C** are very low positive levels. However, Inequality Aversion does not appear particularly compelling. Suppose that **A** is the highest welfare level you can imagine, while **B** and **C** are very low positive levels, **B** being merely slightly higher than **C**. The only difference between a **B**- and a **C**-life, let us assume, is that a **B**-life contains one extra second of mild pleasure. Inequality Aversion implies that if a population at level **C** is large enough, it is improved at least as much by giving everyone an extra second of mild pleasure, as by raising a great number of people to level **A**. This is surely contestable.

 In all events, “Inequality Aversion” is a misnomer. If this condition should for some reason be accepted, this cannot essentially have to do with the badness of inequality. Standard, Archimedean total utilitarianism, according to which the total welfare in a population is measured on a real-valued ratio scale, satisfies Inequality Aversion. Since only the total sum of welfare matters for the value of a population, according to total utilitarianism, this theory is “inequality neutral”. It neither favours nor disfavours inequality in the distribution of welfare. Non-Archimedean totalism is inequality neutral in exactly the same way. This theory, too, ranks populations exclusively by their total sum of welfare. Hence, it is no less “inequality averse” than standard total utilitarianism.[[39]](#footnote-39)

 Actually, Arrhenius acknowledges that Inequality Aversion is intuitively debatable.[[40]](#footnote-40) He nevertheless defends this adequacy condition, by arguing that it follows from the allegedly more compelling Non-Elitism condition.[[41]](#footnote-41) His proof of this implication relies, however, on the Crucial Assumption.[[42]](#footnote-42) Without this assumption, Non-Elitism does not imply Inequality Aversion, as is evident from the fact that non-Archimedean totalism satisfies Non-Elitism but not Inequality Aversion. The Crucial Assumption thus plays an important role also as regards the second and third theorems.

 In addition to his axiological theorems, Arrhenius proves two normative impossibility theorems.[[43]](#footnote-43) Roughly, these theorems are designed to show that there are possible situations of choice, with populations as options, in which the agent cannot avoid choosing wrongly. The following condition is an adequacy condition in both of the normative theorems:

*Normative Inequality Aversion:* For any triplet of welfare levels **A**, **B**, and **C**, **A** higher than **B**, and **B** higher than **C**, and for any population *A* with welfare **A**, there is a larger population *C* with welfare **C**, such that if it is wrong in a certain situation to choose a perfectly equal population *B* of the same size as *A*∪*C* and with welfare **B**, then it is also wrong in the same situation to choose *A*∪*C*, other things being equal.[[44]](#footnote-44)

If non-Archimedean totalism is combined with the normative principle that one ought always to maximize welfare, the resulting theory violates Normative Inequality Aversion. However, this condition is hardly more compelling than Inequality Aversion. If **A** is a very high level, while **B** and **C** are very low positive levels, one can reasonably claim that one ought to raise a large number of people from level **C** to level **A**, rather than raising an even larger number of people from level **C** to level **B**. If so, choosing *B* is wrong, but choosing *A*∪*C* is not.

 As in the case of its axiological cousin, moreover, Normative Inequality Aversion does not really seem to concern inequality. Standard total utilitarianism satisfies this condition, although it is no more inequality averse than non-Archimedean total utilitarianism.

**8. Conclusion**

Four of Arrhenius’s six axiological impossibility theorems presuppose that the order of welfare levels satisfies Archimedeanness. Without this assumption, there are counterexamples to the theorems, in the form of population axiologies satisfying all of Arrhenius’s adequacy conditions. The remaining two theorems also rely on Archimedeanness, albeit less directly. Many philosophers have made claims that are incompatible with Archimedeanness, and non-Archimedean theories of welfare do not seem implausible. Hence, Arrhenius’s theorems rest on controversial assumptions, and the prospects for finding an acceptable population axiology may not be quite as bleak as he argues.[[45]](#footnote-45)

 This does not detract from the significance of Arrhenius’s work. By revealing a great number of inconsistencies or tensions among intuitively very plausible claims, his results considerably restrict the room for maneuver, in the search for a satisfactory population axiology.[[46]](#footnote-46)

**Appendix: Non-Archimedean Totalism and Arrhenius’s First to Fifth Theorems**

We shall verify that non-Archimedean totalism satisfies the adequacy conditions of Arrhenius’s first, fourth, and fifth theorems. The first theorem is as follows:

*The First Impossibility Theorem:* There is no population axiology which satisfies Egalitarian Dominance, Quality, and Quantity.

As compared to the sixth theorem, two of these adequacy conditions are new:

*Quality:* There is a perfectly equal population with very high positive welfare which is at least as good as any population with very low positive welfare, other things being equal.

*Quantity:* For any pair of positive welfare levels **A** and **B**, such that **B** is slightly lower than **A**, and for any number of lives *n*, there is a greater number of lives *m*, such that a population of *m* people at level **B** is at least as good as a population of *n* people at level **A**, other things being equal.

 Non-Archimedean totalism obviously satisfies Quality. To verify Quantity, let levels **A** and **B** be represented by (*h***A**, *l***A**) and (*h***B**, *l***B**), respectively. **A** and **B** being positive, and **B** being slightly lower than **A** implies that *h***A** *=* *h***B** ≥ 0, and *l***A** > *l***B**. Further, if *h***A** *=* *h***B** = 0, then *l***B** > 0. Hence, *n* and *m* can be chosen so that *mh***B** > *nh***A**, or *mh***B** = *nh***A** and *ml***B** ≥ *nl***A**, implying that the population at level **B** is at least as good as the one at level **A**.

 Consider next the fourth theorem:

*The Fourth Impossibility Theorem:* There is no population axiology which satisfies Egalitarian Dominance, General Non-Extreme Priority, Non-Elitism, Weak Non-Sadism, and Quality Addition.

In this theorem, there is only one new condition:

*Quality Addition:* For any population *X*, there is a perfectly equal population with very high welfare, such that its addition to *X* is at least as good as the addition of any population with very low positive welfare to *X*, other things being equal.

 To see that non-Archimedean totalism satisfies Quality Addition, let (*hX*, *lX*) be the total welfare in *X*. Adding a population with very high welfare means adding (*x*, *y*), *x* > 0, to (*hX*, *lX*), whereas adding a population with very low positive welfare means adding (0, *z*), *z* > 0. This implies that (*hX* + *x*, *lX* + *y*) is greater than (*hX*, *lX* + *z*), and hence that the former addition is better than the latter.

 Let us now turn to the fifth theorem:

*The Fifth Impossibility Theorem:* There is no population axiology which satisfies Dominance Addition, Egalitarian Dominance, General Non-Extreme Priority, General Non-Elitism, and Weak Quality.

Three of these adequacy conditions are new:

*Dominance Addition:* An addition of lives with positive welfare and an increase in the welfare of the rest of the population does not make a population worse, other things being equal.

*General Non-Elitism:* For any triplet of welfare levels **A**, **B**, and **C**, **A** slightly higher than **B**, and **B** higher than **C**, and for any one-life population *A* with welfare **A**, there is a population *C* with welfare **C**, and a population *B* of the same size as *A*∪*C* and with welfare **B**, such that for any population *X*, *B*∪*X* is at least as good as *A*∪*C*∪*X*, other things being equal.

*Weak Quality:* There is a perfectly equal population with very high welfare, a very negative welfare level, and a number of lives at this level, such that the high welfare population is at least as good as any population consisting of the lives with negative welfare and any number of lives with very low positive welfare, other things being equal.

 That non-Archimedean totalism satisfies Dominance Addition is obvious. In section 5, we showed that it satisfies Non-Elitism. Since that proof is independent of the welfare levels in *X*, it also proves that General Non-Elitism is satisfied. To see that Weak Quality is satisfied, let (*hA*, *lA*), be the total welfare in a population *A* with very high welfare. It follows that *hA* > 0. The total welfare in a population *B* of lives with very negative welfare is (*hB*, *lB*), *hB* < 0. Adding a number of lives with very low positive welfare to *B*, yields a population *C* witha total welfare of (*hB* + 0, *lB* + *x*), *x* > 0. Since *hA* > *hB*, (*hA*, *lA*) is greater than (*hB* + 0, *lB* + *x*). Hence, *A* is better than *C*.

 Let us finally check that non-Archimedean totalism satisfies all the adequacy conditions of Arrhenius’s second and third theorems, except Inequality Aversion. The second theorem is this:

*The Second Impossibility Theorem:* There is no population axiology which satisfies Dominance Addition, Egalitarian Dominance, Inequality Aversion, and Quality.

 As already noted, it is obvious that non-Archimedean totalism satisfies these conditions, with the exception of Inequality Aversion.

 It remains to consider the third theorem:

*The Third Impossibility Theorem:* There is no population axiology which satisfies Egalitarian Dominance, Inequality Aversion, Non-Extreme Priority, Non-Sadism, and Quality Addition.

Two of these adequacy conditions are new:

*Non-Extreme Priority:* There is a number *n* of lives such that for any population *X*, a population consisting of the *X*-lives, *n* lives with very high welfare, and one life with slightly negative welfare, is at least as good as a population consisting of the *X*-lives and *n* + 1 lives with very low positive welfare, other things being equal.

*Non-Sadism:* [For any population *X*] an addition of any number of people with positive welfare [to *X*] is at least as good as an addition of any number of lives with negative welfare [to *X*], other things being equal.

 That non-Archimedean totalism satisfies Non-Sadism is evident. To verify Non-Extreme Priority, let (*hX*, *lX*) be the total welfare in *X*. Adding *n* lives with very high welfare to *X* means adding (*x*, *y*), *x* > 0, to (*hX*, *lX*), yielding at total welfare of (*hX* + *x*, *lX* + *y*). Then adding one life with slightly negative welfare yields (*hX* + *x* + 0, *lX* + *y* + *z*), *z* < 0. Instead adding *n* + 1 lives with very low positive welfare to *X* yields (*hX* + 0, *lX* + *u*), *u* > 0. Since *x* > 0, (*hX* + *x* + 0, *lX* + *y* + *z*) is greater than (*hX* + 0, *lX* + *u*), implying Non-Extreme Priority.

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1. Parfit, 1984, chapter 19. [↑](#footnote-ref-1)
2. See, e.g., Ng, 1989, Blackorby and Donaldson, 1991, Carlson, 1998. [↑](#footnote-ref-2)
3. For recent statements of Arrhenius’s results, and references to his earlier work, see Arrhenius, 2011, and forthcoming. (Page references to the latter work are to a draft dated September 2012.) [↑](#footnote-ref-3)
4. Arrhenius, 2011, p. 23, forthcoming, p. 378. [↑](#footnote-ref-4)
5. Or at least to all the theorems presented in Arrhenius, forthcoming. [↑](#footnote-ref-5)
6. Arrhenius, 2011, pp. 7-9, forthcoming, pp. 388-391. The purpose of the “other things being equal”-clauses is to leave room for the possibility that other factors than welfare are relevant to the value of a population. [↑](#footnote-ref-6)
7. Arrhenius, 2011, p. 9, forthcoming, p. 339. The label the “sixth” axiological impossibility theorem is from the latter work. The proof is found in Arrhenius, 2011, pp. 9-22, forthcoming, pp. 339-346. [↑](#footnote-ref-7)
8. Arrhenius, 2011, pp. 4-6, forthcoming, pp. 289-291. I have slightly reformulated some of the assumptions. The General Non-Extreme Priority condition seems to require that there is no highest welfare level, and hence that *L* and **L** are at least countably infinite. [↑](#footnote-ref-8)
9. Arrhenius, 2011, p. 5, forthcoming, p. 291. I have simplified Arrhenius’s formulation. [↑](#footnote-ref-9)
10. Arrhenius, 2011, p. 6, forthcoming, p. 291. To accommodate the possibility that (**L**, **≿**) is incomplete, denseness should be defined as requiring only that there is a welfare level between any pair of distinct and *comparable* welfare levels. The existence of a welfare level between any pair of distinct levels obviously implies completeness. [↑](#footnote-ref-10)
11. Arrhenius, 2011, pp. 5-6, forthcoming, p. 291. [↑](#footnote-ref-11)
12. Arrhenius does not explicitly make this comprehensiveness assumption, but it is implicit in his discussion, as well as necessary for his proof. [↑](#footnote-ref-12)
13. If (**L**, **≿**) is itself fine-grained and A-discrete, the Crucial Assumption follows trivially. [↑](#footnote-ref-13)
14. Moreover, it must hold that *f*(**X**) > 0 iff **X** is positive, *f*(**X**) < 0 iff **X** is negative, and *f*(**X**) = 0 iff **X** is neutral. [↑](#footnote-ref-14)
15. Arrhenius, 2011, p. 6, forthcoming, p. 292. Notation slightly altered. [↑](#footnote-ref-15)
16. Although a dense order of welfare levels can have holes, it cannot have gaps, if a gap is defined as a (non-maximal) level lacking a merely slightly higher level. While having no gaps seems sufficient for an A-discrete order to be fine-grained, it is not sufficient in the case of dense orders. [↑](#footnote-ref-16)
17. See, e.g., Luce et al., 2007, p. 49f. [↑](#footnote-ref-17)
18. It is perhaps not very plausible to suggest that Dedekind completeness is a *necessary* condition for fine-grainedness, as regards dense orders. Suppose that (**L**, **≿**) is isomorphic to the rational numbers, naturally ordered, and that differences between welfare levels are ordered in accordance with differences between the representing numbers. In this case, (**L**, **≿**) intuitively seems fine-grained, although it is not Dedekind complete. [↑](#footnote-ref-18)
19. Cf. Luce et al., 2007, p. 50. [↑](#footnote-ref-19)
20. If (**L**, **≿**) is dense and Dedekind complete, what is required for it to fulfil the Crucial Assumption? A sufficient condition, I conjecture, is that it has a countable suborder (**L\***, **≿\***) which is “order dense” in (**L**, **≿**). Order denseness means that for every **X** and **Y** in (**L**, **≿**), such that **X** **≻** **Y**, there is a **Z** in (**L\***, **≿\***), such that **X** **≿** **Z** **≿** **Y**. The real numbers have such a countable order dense subset, viz., the rational numbers, while the lexicographic order of *Re* × *Re* lacks a countable order dense subset. (See Krantz et al., 2007, p. 38ff, Roberts, 2009, p. 111f.) [↑](#footnote-ref-20)
21. Arrhenius, 2011, p. 6. Arrhenius, forthcoming, p. 291f, contains a nearly identical passage. [↑](#footnote-ref-21)
22. I take it that this notion, rather than A-discreteness, corresponds to the standard mathematical meaning of “discrete”. [↑](#footnote-ref-22)
23. Note that S-discreteness and denseness are not jointly exhaustive possibilities. Suppose, for example, that (**L**, **≿**) is isomorphic to the following infinite order of rational numbers: -1, -1/2, -1/3, -1/4, …, 0, …, 1/4, 1/3, 1/2, 1. This order is obviously not dense. Nor is it S-discrete, since 0 has no next greater (or next smaller) number in the order. [↑](#footnote-ref-23)
24. Furthermore, Arrhenius does not argue for the assumption that (**L**, **≿**) is fine-grained. Some adherents of non-Archimedean theories of welfare (see the next section) may want to deny fine-grainedness, and hence the Crucial Assumption. They may argue, for example, that (**L**, **≿**) is dense but contains holes. [↑](#footnote-ref-24)
25. This may imply, of course, that “merely slight” is used in a way that differs from how it is used in ordinary speech. [↑](#footnote-ref-25)
26. This assumption introduces a certain degree of “cardinality” into the framework, by presupposing that differences between welfare levels are to some extent comparable. (The term “roughly equal” is meant to allow, though, for some degree of indeterminacy or incommensurability.) However, this presupposition is implicit already in Arrhenius’s assumption that some differences between levels are “merely slight”. Hence, he is not entirely correct in claiming that his “conditions and theorems only presuppose that lives are quasi-ordered by the relation ‘has at least as high welfare as’”. (Arrhenius, 2011, p. 24.) [↑](#footnote-ref-26)
27. Archimedeanness is very similar to a standard Archimedean axiom in the theory of difference measurement, stating that every “strictly bounded standard sequence” is finite. See Krantz et al., 2007, p. 147, Roberts, 2009, p. 137. [↑](#footnote-ref-27)
28. Parfit, 1986, p. 161; italics in the original. [↑](#footnote-ref-28)
29. I interpret Parfit as claiming that the century of ecstasy contains *more welfare* than the drab eternity. In other words, the welfare level of the former life is higher than the level of the latter life. Parfit could perhaps also be interpreted as holding that the drab eternity contains more welfare than the century of ecstasy, but that the latter life is nevertheless *better* for the person living it. On this interpretation, his view need not violate Archimedeanness with respect to welfare levels. However, it does not matter for our purposes whether or not Parfit, in particular, holds a non-Archimedean view. The important point is that such views are possible and at least *prima facie* plausible. [↑](#footnote-ref-29)
30. Griffin, 1986, p. 85. [↑](#footnote-ref-30)
31. Griffin, 1986, p. 86. [↑](#footnote-ref-31)
32. Indeed, he cites Parfit and Griffin, as well as several other authors (Arrhenius, 2005, p. 98, forthcoming, p. 130), and he even refers to these theories as “non-Archimedean”, albeit in a sense that does not correspond exactly to the denial of Archimedeanness, as stated above. [↑](#footnote-ref-32)
33. Like Arrhenius, I insert an “other things equal”-clause in order not to rule out that other things than welfare can affect the value of a population. [↑](#footnote-ref-33)
34. Arrhenius (2005, p. 106) labels this kind of trumping “strong superiority” between welfare components. Further, he defines “weak superiority” as, roughly, the view that *some* amount of the superior welfare components trumps any amount of the inferior components. He is inclined to reject both forms of superiority, since they have implications he finds counterintuitive. However, these implications follow only under the assumption that the relevant welfare components, or composites of such components, can be ordered in a finite sequence, such that the difference in contributive value to a person’s welfare, between any two adjacent members of the sequence, is “marginal”. (Arrhenius, 2005, p. 107, forthcoming, p. 400.) Arrhenius finds this assumption plausible, but it threatens to beg the question to presuppose it in an argument against superiority, since it is exactly the kind of Archimedean principle that believers in superiority will deny. (Note its similarity to the Crucial Assumption.) [↑](#footnote-ref-34)
35. According to some superiority views, there may be three or more kinds of welfare components, such that the first kind is superior to the second, the second is superior to the third, and so on. Such a view can be represented in terms of ordered *n*-tuples of numbers, instead of ordered pairs. For a general development of this idea, not restricted to the measurement of welfare, see Carlson, 2010. [↑](#footnote-ref-35)
36. In other words, the claim that **X***\** **≻** **X***i*, for any *i*, is compatible with Archimedeanness only if the marginal value of seconds of mild pleasure “converges to a finite limit”, in the sense that, for any difference *d* in (**L**, **≿**), there is a positive integer *j*, such that, for any *k* > *j*, the difference between **X***k* and **X***j* is smaller than *d*. [↑](#footnote-ref-36)
37. Arrhenius, forthcoming, chapter 11. [↑](#footnote-ref-37)
38. Arrhenius, forthcoming, p. 299f. [↑](#footnote-ref-38)
39. This is pointed out by Teru Thomas (n.d.), p. 7. [↑](#footnote-ref-39)
40. Arrhenius, forthcoming, p. 146f. [↑](#footnote-ref-40)
41. Arrhenius, forthcoming, p. 150. [↑](#footnote-ref-41)
42. The proof is on pp. 311-314 in Arrhenius, forthcoming. [↑](#footnote-ref-42)
43. Arrhenius, forthcoming, pp. 366-376. [↑](#footnote-ref-43)
44. Arrhenius, forthcoming, p. 368. [↑](#footnote-ref-44)
45. Some philosophers have responded to population-ethical impossibility results by accepting various versions of Parfit’s “repugnant conclusion” (1984, p. 388). In the case of Arrhenius’s sixth theorem, this would correspond to rejecting Weak Quality Addition. In my opinion, this is a very implausible response. Sometimes, the rejection of adequacy conditions excluding versions of the repugnant conclusion is defended on the grounds that accepting these conditions would force us to deny some other, at least as compelling adequacy condition. (Holtug, 2004, Huemer, 2008, Tännsjö, 2002.) As we have seen, accepting a non-Archimedean theory of welfare allows us to escape this dilemma, at least as it pertains to Arrhenius’s theorems. [↑](#footnote-ref-45)
46. When this draft was almost completed, John Broome sent me an unpublished paper by Teru Thomas (n.d.), containing criticism of Arrhenius’s impossibility theorems that overlaps significantly with mine. In particular, Thomas points out that “lexic utilitarianism”, a population axiology structurally similar to non-Archimedean totalism, violates A-discreteness of (**L**, **≿**). He also states, without proof, that lexic utilitarianism satisfies all the adequacy conditions of Arrhenius’s first, fourth, fifth, and sixth theorems. Further, Thomas remarks that Arrhenius’s proof that Non-Elitism implies Inequality Aversion presupposes A-discreteness. (This is not quite accurate, since the proof merely requires the logically weaker Crucial Assumption.) [↑](#footnote-ref-46)