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## Figure the 6th represents,

A A, The Kidneys.

a a, The Renes Succenturiati.

bb, The Vreters.

c c, The Crural Veins and Arteries.

D, Arteria Magna.

e, Vena Cava.

F, The Bladder.

g g, The Spermatick Vessels, Veins and Arteries.

hh, The Testes, with the Branches of the Veins and Arteries.

i i, The Epididymides.

k k, The Deferentia.

l, The Penis.

m m, Vesiculæ Seminales.

n, Two Glands, from whence a thick Juyce might be press'd out.

o, The Balanus.

An Estimate of the Degrees of the Mortality of Mankind, drawn from curious Tables of the Births and Funerals at the City of Breslaw; with an Attempt to ascertain the Price of Annuities upon Lives. By Mr. E. Halley, R.S.S.

HE Contemplation of the Mortality of Mankind, has besides the Moral, its Physical and Political Uses, both which have been some years since most judiciously considered by the curious Sir William Petty, in his Natural and Political Observations on the Bills of Mortality of London, owned by Captain John Graunt. And since in a like Treatise on the Bills of Mortality of

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Dublin. But the Deduction from those Bills of Mortality seemed even to their Authors to be desective: First, In that the Number of the People was wanting. Secondly, That the Ages of the People dying was not to be had. And Lastly, That both London and Dublin by reason of the great and casual Accession of Strangers who die therein, (as appeared in both, by the great Excess of the Funerals above the Births) rendred them incapable of being Standards for this purpose; which requires, if it were possible, that the People we treat of should not at all be changed, but die where they were born, without any Adventitious Increase from Abroad, or Decay by Migration elsewhere.

This Defect feems in a great measure to be satisfied by the late curious Tables of the Bills of Mortality at the City of Breslaw, lately communicated to this Honourable Society by Mr. Justell, wherein both the Ages and Sexes of all that die are monthly delivered, and compared with the number of the Births, for Five Years last past, viz. 1687, 88, 89, 90, 91, seeming to be done with all the Exactness and Sincerity pos-

fible.

This City of Breslaw is the Capital City of the Province of Silesia; or, as the Germans call it, Schlesia, and is scituated on the Western Bank of the River Oder, anciently called Viadrus; near the Confines of Germany and Poland, and very nigh the Latitude of London. It is very far from the Sea, and as much a Mediterranean Place as can be desired, whence the Consucace of Strangers is but small, and the Manusacture of Linnen employs chiefly the poor People of the place, as well as of the Country round about; whence comes that sort of Linnen we usually call your Sclesse Linnen; which is the chief, if not the only Merchandize of the place. For these Reasons the People of this City seem most pro-

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per for a Standard; and the rather, for that the Births do, a small matter, exceed the Funerals. The only thing wanting is the Number of the whole People, which in some measure I have endeavoured to supply by comparison of the Mortality of the People of all Ages, which I shall from the said Bills trace out with

all the Acuracy possible.

It appears that in the Five Years mentioned, viz. from 87 to 91 inclusive, there were born 6193 Persons, and buried 5869; that is, born per Annum 1238, and buried 1174; whence an Encrease of the People may be argued of 64 per Annum, or of about a 20th part, which may perhaps be ballanced by the Levies for the Emperor's Service in his Wars. But this being contingent, and the Births certain, I will suppose the People of Breslaw to be encreased by 1238 Births annually. Of these it appears by the same Tables, that 348 do die yearly in the first Year of their Age, and that but 890 do arrive at a full Tears Age; and likewise, that 198 do die in the Five Years between 1 and 6 compleat, taken at a Medium; so that but 692 of the Persons born do survive Six whole Tears. From this Age the Infants being arrived at some degree of Firmness, grow less and less Mortal; and it appears that of the whole People of Breslaw there die yearly, as in the following Table, wherein the upper Line shews the Age, and the next under it the Number of Persons of that Age dying yearly.

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7.89.14.18.21.27.28.35.
11.11.6.
$$5\frac{1}{2} \cdot 2 \cdot 3\frac{1}{2} \cdot 5 \cdot 6 \cdot 4\frac{1}{2} \cdot 6\frac{1}{2} \cdot 9 \cdot 8 \cdot 7 \cdot 7$$
.

36. 42. 45. 49.54.55.56.63.
8.9\frac{1}{2} \text{8.9.} 7.7.10.11.9.9.10.12.

70.71.72. 77.81.84.90.91.
9\frac{1}{2} \text{14.9.11.9\frac{1}{2}} \text{6.7.3.4.2.1.1.1.1.}

And where no Figure is placed over, it is to be underflood of those that die between the Ages of the preceding and consequent Column.

From this Table it is evident, that from the Age of 9 to about 25 there does not die above 6 per Annum of each Age, which is much about one per Cent. of those that are of those Ages: And whereas in the 14, 15,16, 17 Tears there appear to die much fewer, as 2 and 31, yet that seems rather to be attributed to Chance, as are the other Irregularities in the Series of Ages, which would rectifie themselves, were the number of Years much more considerable, as 20 instead of 5. And by our own Experience in Christ-Church Hospital, I am informed there die of the Toung Lads, much about one per Cent. per Annum, they being of the foresaid Ages. From 25 to 50 there seem to die from 7 to 8 and 9 per Annum of each Age; and after that to 70, they growing more crafte, though the number be much diminished, yet the Mortality encreases, and there are found to die 10 or 11 of each Age per Annum: From thence the number of the Living being grown very small, they gradu(600)

gradually decline till there be none left to die; as may be seen at one View in the Table.

From these Considerations I have formed the adjoyned Table, whose Uses are manifold, and give a more just Idea of the State and Condition of Mankind, than any thing yet extant that I know of. It exhibits the Number of People in the City of Breslaw of all Ages, from the Birth to extream Old Age, and thereby shews the Chances of Mortality at all Ages, and likewise how to make a certain Estimate of the value of Annuities for Lives, which hitherto has been only done by an imaginary Valuation: Also the Chances that there are that a Person of any Age proposed does live to any other Age given; with many more, as I shall hereaster shew. This Table does shew the number of Persons that are living in the Age current annexed thereto, as follows:

Age.	Per-	Age.	Per-	Age.	Per-	Age.	Per-	Age	Per-	Age	Pet-	Age.	Persons.
Curt.	fons.	Curt.	fons	Curt.	fons	Curt.	fons	Curt.	fons	Curt	fons	1 -	
1	1000		580	, ,	628		585	29	539	35	481	7	<b>5</b> 54 <b>7</b>
2	855	9	670		622	23	579	30	531	37	472		4584
3	798		651	/	616		573	31	523	38	463		4270
4	760	H	653		610		567	32	515	39	454		3964 3604
5	732	12	546		60.4		560		507	40	145	42	3178
6	710		640	20	598		553		499		430	49	2709
7	692		634		592	28	546		490	42	427	55	2194
1 6 7	Per-	Age.	Per-	Age.	Per-	ge	Per-	Age.	Per-	Age.	Per-	63	1694
{	fons.				-	-		Curt.	ions	Urir.	ions	7ó	1204
43	417	-	340	57	272	64	202	7 I	131	78	58	77	692
44	407	51	335		262	65	192	72	120	79	49	84	253
45	397	52	324		252	65	182	73	109	80	• ;	100	107
46	387	53	313		242	67	172	74	98	18	34		
47	377	1	302		232	68	162	75	88	82	28		34000
48	357		292 282		222	69	152	76	78	83	23		
1 47	95/1	27 1	2021	- 4g	2 1 2	70	142	77	631	84	201	Sun	Total.

Thus it appears, that the whole People of Breslaw does consist of 34000 Souls, being the Sum Total of the Persons of all Ages in the Table: The first wie hereof

is to shew the Proportion of Men able to bear Arms in any Multitude, which are those between 18 and 56, rather than 16 and 60; the one being generally too weak to bear the Fatigues of War and the Weight of Arms, and the other too crasse and insirm from Age, notwithstanding particular Instances to the contrary. Under 18 from the Table, are sound in this City 11997 Persons, and 3950 above 56, which together make 15947. So that the Residue to 34000 being 18053 are Persons between those Ages. At least one half thereof are Males, or 9027: So that the whole Force this City can raise of Fencible Men, as the Scotch call them, is about 9000, or 34, or somewhat more than a quarter of the Number of Souls, which may perhaps pass for a Rule for all other places.

The Second Use of this Table is to shew the differing degrees of Mortality, or rather Vitality in all Ages; for if the number of Persons of any Age remaining after one year, be divided by the difference between that and the number of the Age proposed, it shews the odds that there is, that a Person of that Age does not die in a Tear. As for Instance, a Person of 25 Tears of Age has the odds of 560 to 7 or 80 to 1, that he does not die in a Tear: Because that of 567, living of 25 years of Age, there do die no more than 7 in a Tear, leaving

560 of 26 Years old.

So likewise for the odds, that any Person does not die before he attain any proposed Age: Take the number of the remaining Persons of the Age proposed, and divide it by the difference between it and the number of those of the Age of the Party proposed; and that shews the odds there is between the Chances of the Party's living or dying. As for Instance; What is the odds that a Man of 40 lives 7 Years: Take the number of Persons of 47 years, which in the Table is 377, and sub-

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substract it from the number of Persons of 40 years, which is 445, and the difference is 68: Which shews that the Persons dying in that 7 years are 68, and that it is 377 to 68 or 5½ to 1, that a Man of 40 does live 7 Years. And the like for any other number of Tears.

Tears, it is an even Lay that a Person of any Age shall die, this Table readily persorms it: For if the number of Persons living of the Age proposed be halfed, it will be found by the Table at what Year the said number is reduced to half by Mortality; and that is the Age, so which it is an even Wager, that a Person of the Age proposed shall arrive before he die. As for Instance; A Person of 30 Years of Age is proposed, the number of that Age is 531, the half thereof is 265, which number I find to be between 57 and 58 Years; so that a Man of 30 may reasonably expect to live between 27 and 28 Years.

rance upon Lives ought to be regulated, and the difference is discovered between the price of ensuring the Life of a Man of 20 and 50, for Example: it being 100 to 1 that a Man of 20 dies not in a year, and but

38 to 1 for a Man of 50 Years of Age.

Use V. On this depends the Valuation of Annuities upon Lives; for it is plain that the Purchaser ought to pay for only such a part of the value of the Annuity, as he has Chances that he is living; and this ought to be computed yearly, and the Sum of all those yearly Values being added together, will amount to the value of the Annuity for the Life of the Person proposed. Now the present value of Money payable after a term of years, at any given rate of Interest, either may be had from Tables already computed; or almost as compendicusly,

by

( 603 )

by the Table of Logarithms: For the Arithmetical Complement of the Logarithm of Unity and its yearly Interest (that is, of 1, 06 for Six per Cent. being 9, 974694.) being multiplied by the number of years propoled, gives the present value of One Pound payable after the end of fo many years. Then by the foregoing Proposition, it will be as the number of Perions living after that term of years, to the number dead; so are the Odds that any one Person is Alive or Dead. And by consequence, as the Sum of both or the number of Persons living of the Age first proposed, to the number remaining after to many years, (both given by the Table) so the present value of the yearly Sum payable aster the term proposed, to the Sum which ought to be paid for the Chance the person has to enjoy such an Annuity after so many Years. And this being repeated for every year of the persons Life, the Sum of all the present Values of those Chances is the true Value of the Annuity. This will without doubt appear to be a most laborious Calculation, but it being one of the principal Uses of this Speculation, and having found some Compendia for the Work, I took the pains to compute the following Table, being the short Result of a not ordinary number of Arithmetical Operations; It shews the Value of Annuities for every Fifth Year of Age, to the Seventieth, as follows.

Age.	Years Purchase.	Age.	Years Purchase.	Age.	Years Purchase.
1	10,28	25	12,27	50	9,21
5	13,40	30	11,72	55	8,5 r
10	13,44	35	11,12	60	7,60
15	13,33	40	10,57	65	6,54
20	12,78	45	9, <b>91</b>	70	5,32

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This shews the great Advantage of putting Money into the present Fund lately granted to their Majesties, giving 14 per Cent. per Annum, or at the rate of 7 years purchase for a Life; when young Lives, at the usual rate of Interest, are worth above 13 years Purchase. It shews likewise the Advantage of young Lives over those in Years; a Life of Ten Years being almost worth 13; years purchase, whereas one of 36 is worth but 11.

Use V. Two Lives are likewise valuable by the same Rule; for the number of Chances of each fingle Life, found in the Table, being multiplied together, become the Chances of the Two Lives. And after any certain Term of Years, the Product of the two remaining Sums is the Chances that both the Persons are living. The Product of the two Differences, being the numbers of the Dead of both Ages, are the Chances that both the Perfons are dead. And the two Products of the remaining Sums of the one Age multiplied by those dead of the other, shew the Chances that there are that each Party furvives the other: Whence is derived the Rule to estimate the value of the Remainder of one Life after another. Now as the Product of the Two Numbers in the Table for the Two Ages proposed, is to the difference between that Product and the Product of the two numbers of Persons deceased in any space of time. so is the value of a Sum of Money to be paid after so much time, to the value thereof under the Contingency of Mortality. And as the aforefaid Product of the two Numbers answering to the Ages proposed, to the Product of the Deceased of one Age multiplied by those remaining alive of the other; So the Value of a Sum of Money to be paid after any time proposed, to the value of the Chances that the one Party has that he survives the other whose number of Deceased you made use of, in the second Term of the proportion. This perhaps

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may be better understood, by putting N for the number of the younger Age, and n for that of the Elder; T, y the deceased of both Ages respectively, and R, r for the Remainders; and R+T=N and r+y=n. Then shall N n be the whole number of Chances;  $N_{R}-T_{Y}$  be the Chances that one of the two Persons is living, Ty the Chances that they are both dead; R y the Chances that the elder Person is dead and the younger living; and r T the Chances that the elder is living and the younger dead. Thus two Persons of 18 and 35 are proposed, and after 8 years these Chances are required. The Numbers for 18 and 35 are 610 and 490, and there are 50 of the First Age dead in 8 years, and 73 of the Elder Age. There are in all 610 x 490 or 298900 Chances; of these there are 50x73 or 3650 that they are both dead. And as 298900, to 298900 -3650, or 295250: So is the present value of a Sum of Money to be paid after 8 years, to the present value of a Sum to be paid if either of the two live. 560 x 73, fo are the Chances that the Elder is dead, leaving the Younger; and as 417x50, so are the Chances that the Younger is dead, leaving the Elder. Wherefore as 610x490 to 560x73, so is the present value of a Sum to be paid at eight years end, to the Sum to be paid for the Chance of the Youngers Survivance; and as 610 x 490 to 417 x 50, so is the same present value to the Sum to be paid for the Chance of the Elders Survivance.

This possibly may be yet better explained by expounding these Products by Rectangular Parallelograms, as in Fig. 7. wherein AB or CD represents the number of persons of the younger Age, and DE, BH those remaining alive after a certain term of years; whence CE will answer the number of those dead in that time: So AC, BD may represent the number

 $\mathbf{D}_{2}$ 

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of the Elder Age; A F,B I the Survivors after the same term; and CF, DI, those of that Age that are dead at that time. Then shall the whole Parallelogram ABCD be Nn, or the Product of the two Numbers of persons, representing such a number of Persons of the two Ages given; and by what was faid before, after the Term proposed the Rectangle HD shall be as the number of Persons of the younger Age that survive, and the Rectangle AE as the number of those that die. So likewise the Rectangles AI, FD shall be as the Numbers, living and dead, of the other Age. Hence the Rectangle HI shall be as an equal number of both Ages surviving. The Rectangle FE being the Product of the deceased, or  $\Upsilon y$ , an equal number of both dead. The Rectangle GD or Ry, a number living of the younger Age, and dead of the Elder: And the Rectangle AG or r Y a number living of the Elder Age, but dead of the younger. This being understood, it is obvious, that as the whole Rectangle AD or Nn is to the Gnomon FABDEG or  $Nn-\Upsilon y$ , so is the whole number of Persons or Chances, to the number of Chances that one of the two Persons is living: And as AD or Nn is to FE or  $\Upsilon \gamma$ , so are all the Chances, to the Chances that both are dead; whereby may be computed the value of the Reversion after both Lives. And as AD to GD or Ry, so the whole number of Chances, to the Chances that the younger is living and the other dead; whereby may be cast up what value ought to be paid for the Reversion of one Life after another, as in the case of providing for Clergy-mens Widows and others by such Reversions. And as AD to AG or r Y, so are all the Chances, to those that the Elder survives the younger. I have been the more particular, and perhaps tedious, in this matter, because it is the Key to the Case of Three Lives, which of it felf would not have been so easie to comprehend.

VII. If Three Lives are proposed, to find the value of an Annuity during the continuance of any of those three Lives. The Rule is, As the Product of the continual multiplication of the Three Numbers, in the Table, answering to the Ages proposed, is to the difference of that Product and of the Product of the Three Numbers of the deceased of those Ages, in any given term of Years; So is the present value of a Sum of Money to be paid certainly after so many Years, to the present value of the

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same Sum to be paid, provided one of those three Persons be living at the Expiration of that term. Which proportion being yearly repeated, the Sum of all those present values will be the value of an Annuity granted for three fuch Lives. But to explain this, together with all the Cases of Survivance in three Lives: Let N be the Number in the Table for the Younger Age, n for the Second, and v for the Elder Age; let T be those dead of the Younger Age in the term proposed, y those dead of the Second Age, and v those of the Elder Age; and let R be the Remainder of the younger Age, r that of the middle Age, and g the Remainder of the Elder Age. Then shall R-|-r| be equal to N, r-|-y| to n, and 9 + v to v, and the continual Product of the three Numbers N n r shall be equal to the continual Product of R + r $x \rightarrow x + x + v$ , which being the whole number of Chances for three Lives is compounded of the eight Products following. (1) Rrg, which is the number of Chances that all three of the Persons are living. (2)  $r \in \mathcal{T}$ , which is the number of Chances that the two Elder Persons are living, and the younger dead. (3) Rpy the number of Chances that the middle Age is dead, and the younger and Elder living. (4) Rru being the Chances that the two younger are living, and the elder dead. (5) & Ty the Chances that the two younger are dead, and the elder living. (6)  $r \Upsilon v$  the Chances that the younger and elder are dead, and the middle Age living. (7) Ryu, which are the Chances that the younger is living, and the two other dead. And Lastly and Eightly, Ty v, which are the Chances that all three are dead. Which latter fubstracted from the whole number of Chances Nnv, leaves Nnv-T y v the Sum of all the other Seven Products; in all of which one or more of the three Persons are surviving.

To make this yet more evident, I have added Fig. 8. wherein these Eight several Products are at one view exhibited. Let the rectangled Parallelepipedon ABCDEFGH be constituted of the sides AB, GH, &c. proportional to N the number of the younger Age; AC, BD, &c. proportional to s; and AG, CE, &c. proportional to the number of the Elder, or v. And the whole Parallelepipedon shall be as the Product Nnv, or our whole number of Chances. Let RP be as R, and AP as Y: let CL be as r, and Ln as y; and GN as g, and NA as v; and let the Plain PR e.a be made parallel to the

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plain ACGE; the plain NVbY parallel to ABCD; and the plain LXTQ parallel to the plain ABGH. And our first Product Rrg shall be as the Solid STWIFZeb. The Second, or rg r will be as the Solid EYZe QSMI. The Third, Rey, as the Solid RHOVWIST. And the Fourth, Rru, as the Solid Zab DWXIK. Fifthly, g Ty, as the Solid GQRSIMNO. Sixthly, rYu, as IKLMGYZA. Seventhly. Ryo, as the Solid IKPOBXVW. And Laftly, AIKLMNOP will be as the Product of the 2 numbers of persons dead, or Y y v. I shall not apply this in all the cases thereof for brevity fake; only to shew in one how all the rest may be performed, let it be demanded what is the value of the Reversion of the younger Life after the two elder proposed. The proportion is as the whole number of Chances, or  $N_{n\nu}$ to the Product Ry v, so is the certain present value of the Sum payable after any term proposed, to the value due to fuch Chances as the younger person has to bury both the elder, by the term proposed; which therefore he is to pay for. Here it is to be noted, that the first term of all these Proportions is the same throughout, viz. Nnv. The Second changing yearly according to the Decrease of R, r, g, and  $E_{n-1}$ crease of  $\Upsilon$ ,  $\gamma$ ,  $\nu$ . And the third are successively the present values of Money payable after one, two, three, &c. years, according to the rate of Interest agreed on. These numbers. which are in all cases of Annuities of necessary use, I have put into the following Table, they being the Decimal values of One Pound payable after the number of years in the Margent, at the rate of 6 per Cent.

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Years.	Present va- lue of 1 l.		Present va- lue of 1 l.	Years.	Present va- lue of 1 1.
1	0,9434	19	0,3305	37	0,1158
2	0,8900	20	0,3118	38	0,1092
3	0,8396	21	0,2941	39	0,1031
4	0,7921	22	0,2775	40	0,0972
5	7473د0	23	0,2618	45.	0,0726
6	0,7050	24	0,2470	50	0,0543
7 8	0,6650	25	0,2330	55	0,0406
8	0,6274	26	0,2198	60	0,0303
9	0,5919	27	0,2074	65	0,0227
10	0,5584	28	0,1956	70	0,0169
11	0,5268	29	0,1845	75	0,0126
12	0,4970	30	0,1741	80	0,0094
13	0,4688	3.1	0,1643	85	0,0071
14	4423ر٥	32	0,1550	90	0,0053
15	0:4173	33	0,1462	95	0,0039
16	0,3936	34	0,1379	100	0,0029
17	0,3714	35	0,1301		
18	0,3503	36	0,1227		

It were needless to advertise, that the great trouble of working so many Proportions will be very much alleviated by using Logarithms; and that instead of using  $Nn\nu-Ty\nu$  for the Second Term of the Proportion in finding the value of Three Lives, it may suffice to use only  $Ty\nu$ , and then deducting the Fourth Term so found out of the Third, the Remainder shall be the present value sought; or all these Fourth Terms being added together, and deducted out of the value of the certain Annuity for so many Years, will leave the value of the contingent Annuity upon the Chance of Mortality of all those three Lives. For Example; Let there be Three Lives of 10,30, and 40 years of Age proposed, and the Proportions will be thus:

As 661 in 521 in 445 or 156190995, or N=19
to 8 in 8 in 9, or 576, or Yy 10 for the first year, so 0, 9434. to 0,00000348
to 15 in 16 in 18, or 4320, for the second year, so 0, 8300. to 0,000008123
to 27 in 32 in 38, so 14112 for the third year, so 0, 8396. to 0,00008123
to 27 in 32 in 38, so the forth year, so 0, 7921. to 0,00031071
to 39 in 50 in 58, for the fixth year, so 0, 7050 to 0,00031071
And

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And so forth to the 60th year, when we suppose the elder Life of Forty certainly to be expired; from whence till Seventy we must compute for the First and Second only, and from thence to Ninety for the single youngest Life. Then the Sum Total of all these Fourth Proportionals being taken out of the value of a certain Annuity for 90 Years, being 16,58 years Purchase, shall leave the just value to be paid for an Annuity during the whole term of the Lives of three Persons of the Ages proposed. And note, that it will not be necessary to compute for every year singly, but that in most cases every 4th or 5th year may suffice, interpoling for the intermediate years secundum artem.

It may be objected, that the different Salubrity of places does hinder this Proposal from being universal; nor can it be denied. But by the number that die, being 1174. per Annum in 34000, it does appear that about a 30th part die yearly, as Sir William Petty has computed for London; and the number that die in Infancy, is a good Argument that the Air is but indifferently salubrious. So that by what I can learn, there cannot perhaps be one better place proposed for a Standard. At least 'tis desired that in imitation hereof the Curious in other Cities would attempt something of the same nature, than

which nothing perhaps can be more useful.